Linear mixed effects models

and where to find them

Elena Rybina for ACME

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Naming

Linear mixed effects models

Hierarchical linear models

Nested data models

Multilevel models

Random coefficient

Split-plot designs

Random-effects models

Linear mixed effects models

- Generalization of linear models (linear regression)
- Several IVs (predictors) —> Single DV (response)
- Multilevel IVs (nested data, between or within subjects)
- Fixed effects x random effects

Linear regression

- Y (response) <— X_i (predictors)

Random Error term

Independent Variable

Dependent Variable

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

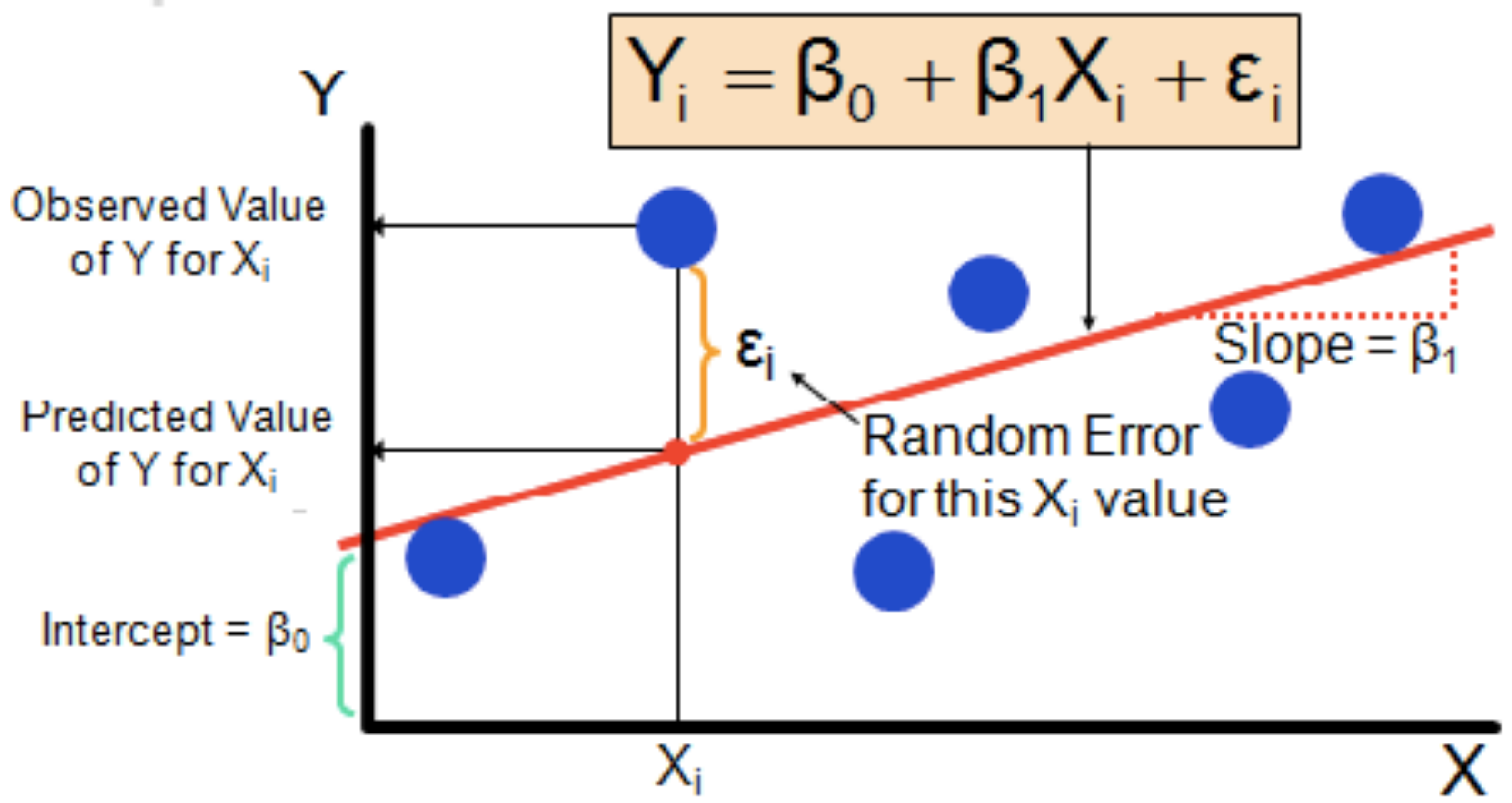
"Fixed effect parameters"

$$\textit{observed data} \rightarrow \quad y = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p + \varepsilon$$

$$\textit{predicted data} \rightarrow \quad y' = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p$$

$$\textit{error} \rightarrow \quad \varepsilon = y - y'$$

Linear regression



Random effect models

- Y (response) <— X_i (predictors)
- Nested (hierarchical) data
- Constant random component

$$y_{ij} = b_0 + b_1 x_{ij} + v_i + e_{ij}$$

"Random effect"

Example:

$$Y_{ij} = \mu + \beta_1 \mathrm{Sex}_{ij} + \beta_2 \mathrm{Race}_{ij} + \beta_3 \mathrm{ParentsEduc}_{ij} + U_i + W_{ij},$$

LME: Fixed + random effects

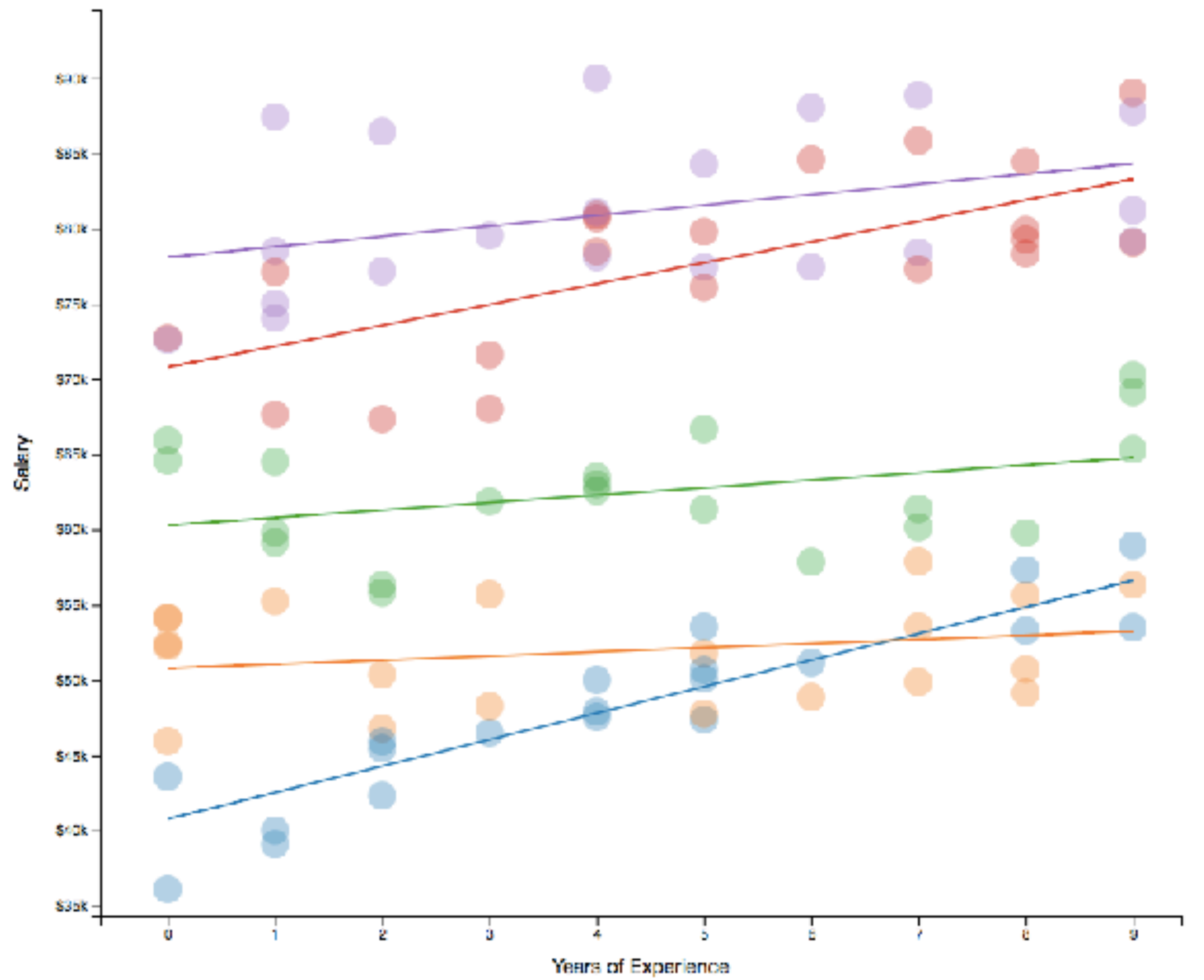
$$y_{ij} = b_0 + b_1 x_{ij} + v_i + e_{ij}$$

for $i \in \{1, \ldots, n\}$ and $j \in \{1, \ldots, m_i\}$ where

- $y_{ij} \in \mathbb{R}$ is the response for j-th measurement of i-th subject
- $b_0 \in \mathbb{R}$ is the fixed intercept for the regression model
- $b_1 \in \mathbb{R}$ is the fixed slope for the regression model
- $x_{ij} \in \mathbb{R}$ is the predictor for j-th measurement of i-th subject
- $v_i \overset{\text{iid}}{\sim} \mathrm{N}(0, \sigma_v^2)$ is the random intercept for the i-th subject
- $e_{ij} \overset{\text{iid}}{\sim} \mathrm{N}(0, \sigma_e^2)$ is a Gaussian error term

Nice example

http://mfviz.com/hierarchical-models/



How to use in R

- Packages ***lme4*** (Linear Mixed Effects), *nlme* (Non-Linear Mixed Effects), *asreml* (average spatial reml)

- Function ***lmer()***

Score ~ Age + ParentsEduc + (1|Subject) + (1| School) + ε

>> lmer(Score ~ Age + ParentsEduc + (1|Subject) + (1| School),

data = datatable)

IOR data

Figure 1. Trial with the valid location of the target

IOR data

```
model1 <- lmer( MRT ~ probeOnset+ (1|NewSubject), dt)

model2 <- lmer( MRT ~ probeOnset+ validy+(1|NewSubject), dt)

model3 <- lmer( MRT ~ probeOnset+ validy+rFrame+(1|NewSubject), dt)

model3a <- lmer( MRT ~ early+ validy+early:validy:rFrame+(1|NewSubject), dt)


resultfull <- lmer( MRT ~ probeOnset*validy*rFrame + (1|NewSubject), dt)
```

Usage

- Nested data
- Repeated measures
- Longitudinal studies

LME vs ANOVA?

- Incorrect question! (ans: both)
- ANOVA can use LME
- *aov()*, *anova(), ezANOVA()*
  aov(Y ~ B + Error(A/B), data=datatable)
- Or use *anova.lme()* from the *nlme* package

Resources

- Nice visualization http://mfviz.com/hierarchical-models/
- R function ***lmer*** good example http://lme4.r-forge.r-project.org/book/Ch1.pdf
- R ***lmer*** nice tutorial http://www.bodowinter.com/tutorial/bw_LME_tutorial2.pdf
- Math staff http://www2.stat.duke.edu/~sayan/Sta613/2017/lec/LMM.pdf